

# Delta-based Verification of Software Product Families



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**Verification**  
of  
**Software Product Lines**  
implemented with  
**Delta Oriented Programming (ABS)**



```
1 class C {  
2   Int f;  
3  
4   Unit m(Int x) {  
5     this.f = this.f + x;  
6   }  
7  
8   Unit m1(Int x) {  
9     this.m(x);  
10    this.f = this.f + 1;  
11  }  
12 }
```

Variants:

{ C }



```
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```
1 delta D1 when Feature1{
2   modifies C {
3     modifies Unit m(Int x) {
4       original(x);
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6     }
7   }
8 }
```

Variants:

{ C, C.D1 }



```
1 class C {  
2   Int f;  
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5     this.f = this.f + x;  
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8   Unit m1(Int x) {  
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```
1 delta D1 when Feature1{  
2   modifies C {  
3     modifies Unit m(Int x) {  
4       original(x);  
5       this.f = this.f + 2;  
6     }  
7   }  
8 }
```

```
1 delta D2 when Feature2{  
2   modifies C {  
3     modifies Unit m(Int x) {  
4       original(x);  
5       this.f = this.f + 3;  
6     }  
7   }  
8 }
```

Variants:

{ C, C.D1, C.D2, C.D1.D2 }

```
1 class C {  
2   Int f;  
3  
4   Unit m(Int x) {  
5     this.f = this.f + x;  
6   }  
7  
8   Unit m1(Int x) {  
9     this.m(x);  
10    this.f = this.f + 1;  
11  }  
12 }
```

```
1 delta D1 when Feature1{  
2   modifies C {  
3     modifies Unit m(Int x) {  
4       original(x);  
5       this.f = this.f + 2;  
6     }  
7   }  
8 }
```

```
1 delta D2 when Feature2{  
2   modifies C {  
3     modifies Unit m(Int x) {  
4       original(x);  
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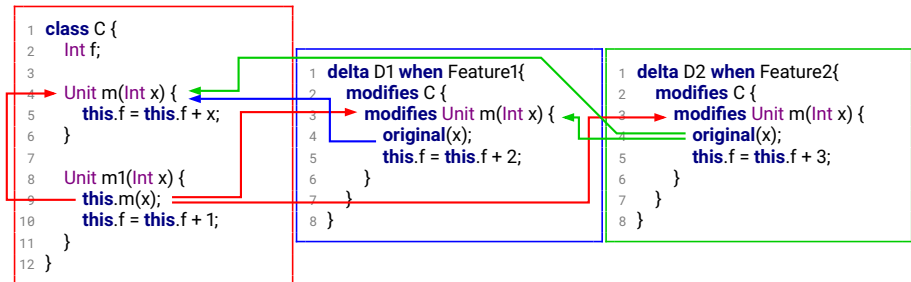
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1 delta D2 when Feature2{  
2   modifies C {  
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6     }  
7   }  
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2    Int f;  
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5    //@ensures this.f = \old(this.f) + x;  
6    //@assignable this.f;  
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8    Unit m(Int x) {  
9      this.f = this.f + x;  
10   }  
11 }  
12
```

Behavior of `m` specified by

(`m.pre`, `m.post`, `m.frame`)

```
1  class C {  
2    Int f;  
3  
4    //@requires True; ← precondition  
5    //@ensures this.f = \old(this.f) + x;  
6    //@assignable this.f;  
7  
8    Unit m(Int x) {  
9      this.f = this.f + x;  
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8    Unit m(Int x) {  
9      this.f = this.f + x;  
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```

Behavior of `m` specified by

(`True`, `m.post`, `m.frame`)

```
1  class C {  
2    Int f;  
3  
4    //@requires True;  
5    //@ensures this.f = \old(this.f) + x; ← postcondition  
6    //@assignable this.f;  
7  
8    Unit m(Int x) {  
9      this.f = this.f + x;  
10   }  
11 }  
12
```

Behavior of `m` specified by

(True, `m.post`, `m.frame`)

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1  class C {  
2    Int f;  
3  
4    //@requires True;  
5    //@ensures this.f = \old(this.f) + x; ← precondition  
6    //@assignable this.f;  
7  
8    Unit m(Int x) {  
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Behavior of `m` specified by

(True, `this.f = \old(this.f) + x`, `m.frame`)

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1  class C {  
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7  
8    Unit m(Int x) {  
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Behavior of `m` specified by

$(\text{True}, \text{this.f} = \text{\old(this.f)} + x, \text{m.frame})$

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1  class C {  
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(True, `this.f = \old(this.f) + x`, `{this.f}`)



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12
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Behavior of `m` specified by

(True, this.f = \old(this.f) + x, {this.f})

**!! We focus on normal behaviors !!**

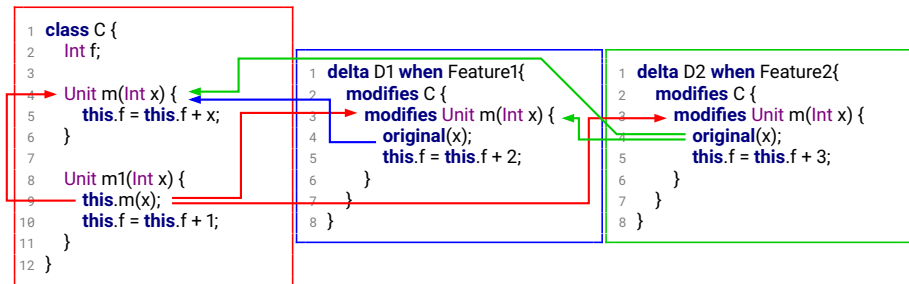


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1 class C {
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8     this.f = this.f + x;
9   }
10
11  //@requires True;
12  //@ensures this.f = \old(this.f) + x + 1;
13  //@assignable this.f;
14  Unit m1(Int x) {
15    this.m(x);
16    this.f = this.f + 1;
17  }
18 }
```

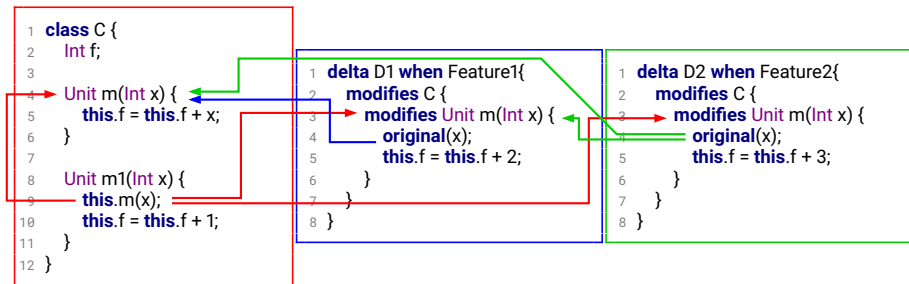
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```

# Compositional Verification ??



Straightforward solution: **product-based** verification



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Expensive: **exponential complexity**  $O(2^N * M)$

(N nr. of deltas, M nr. of methods)



## Liskov Substitution Principle for DOP

Hähnle and Schaefer (2012)



# More-general-than Relation ( $\succ$ )

```
1 class C {
2     Int f;
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10    //...
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```

```
1 delta D1 when Feature1{
2     modifies C {
3         //@requires True;
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```

**C.m.contract** *is-more-general-than* **D1.C.m.contract**

# More-general-than Relation ( $\supseteq$ )

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C.m.contract  $\supseteq$

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D1.C.m.contract

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8             this.f = this.f + 2;
9         }
10    }
11 }
```

$(C.m.preq, C.m.post, C.m.frame) \supseteq (D1.C.m.preq, D1.C.m.post, D1.C.m.frame)$

# More-general-than Relation ( $\succeq$ )



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1 class C {
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$(m.\text{preq}, m.\text{post}, m.\text{frame}) \succeq (m.\text{preq}, m.\text{post}, m.\text{frame})$

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$(m.\text{preq}, m.\text{post}, m.\text{frame}) \supseteq (m.\text{preq}, m.\text{post}, m.\text{frame})$

*if and only if*

$m.\text{preq} \Rightarrow m.\text{preq}$   
 $m.\text{post} \Leftarrow m.\text{post}$   
 $m.\text{frame} \supseteq m.\text{frame}$

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*if and only if*

$\text{True} \Rightarrow \text{True}$   
 $f \geq \text{old}(f) + x \Leftarrow f \geq \text{old}(f) + x + 2$   
 $m.\text{frame} \supseteq m.\text{frame}$



# More-general-than Relation ( $\succeq$ )

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$(m.\text{preq}, m.\text{post}, m.\text{frame}) \succeq (m.\text{preq}, m.\text{post}, m.\text{frame})$

*if and only if*

$\text{True} \Rightarrow \text{True}$   
 $f \geq \text{old}(f) + x \Leftarrow f \geq \text{old}(f) + x + 2$   
 $\{f\} \supseteq \{f\}$

# More-general-than Relation ( $\supseteq$ )

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```

**C.m.contract**  $\supseteq$  **D1.C.m.contract**

then

**D1.m**  
satisfies  
**D1.m.contract**  $\implies$  **D1.m**  
satisfies  
**C.m.contract**

# More-general-than Relation ( $\succ$ )

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A correct method `m` **fulfills every contract more general** than its own

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11 }
```

A correct method `m` **fulfills every contract more general** than its own



When **calling** `m` we can consider its **most general contract**

---

# Liskov Substitution Principle for DOP

Hähle and Schaefer (2012)

---



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For each version of  $m$   
its contract is **more-specific-than each previous one**

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Hähnle and Schaefer (2012)

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TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

For each version of  $m$   
its contract is **more-specific-than each previous one**

+

Each method is **Liskov-verified**:  
*for each method call we consider its **most general contract***

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Every variant **is correct**

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Each method is **Liskov-verified**:  
*for each method call we consider its **most general contract***



Every variant **is correct**

+

**No variant is generated**: polynomial complexity  **$O(N*M)$**   
( $N$ =nr. of deltas,  $M$ =nr. of methods)





Program behaviors can **only** become **more specific**

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All the versions of a method have  
the **same (most general) behavior**

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Deltas **do not modify** method **behaviors**

**Goal:** Overcome Liskov's restrictions *without increasing costs*

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**How?**

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**Represent this set abstractly:**  
*original calls as abstract programs*

**Goal:** Overcome Liskov's restrictions *without increasing costs*

**How?**

An **original call** in  $m$  refers to a **set of versions** of  $m$

+

**Represent this set abstractly:**  
*original calls as abstract programs*



**Verifying each delta in isolation**

*(delta-based verification)*



# How to Deal with Abstract Programs?

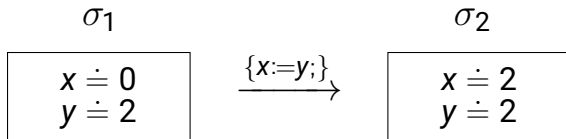
$\sigma_1$

$$\begin{array}{l} x \doteq 0 \\ y \doteq 2 \end{array}$$

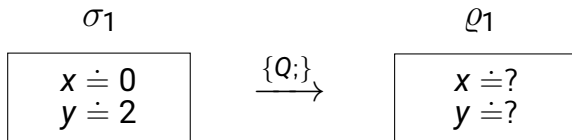
# How to Deal with Abstract Programs?

$$\sigma_1 \quad \boxed{\begin{array}{l} x \doteq 0 \\ y \doteq 2 \end{array}} \quad \xrightarrow{\{x:=y;\}}$$

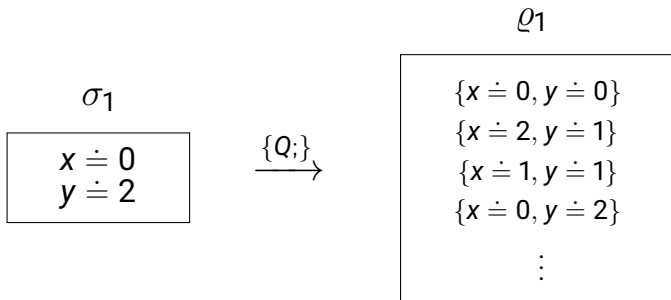
# How to Deal with Abstract Programs?



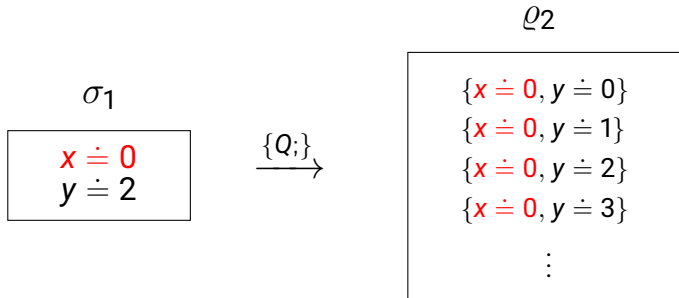
# How to Deal with Abstract Programs?



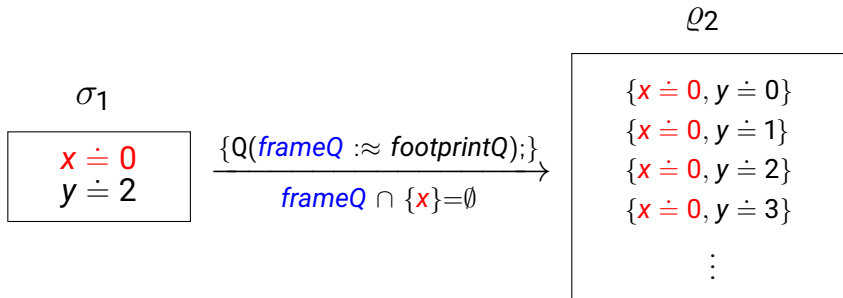
# How to Deal with Abstract Programs?



# How to Deal with Abstract Programs?



If  $Q$  cannot assign  $x$



Q is modelled as an **Abstract Statement**

$\sigma_1$

$x \doteq 0$   
 $y \doteq 2$

$\frac{\{Q(\text{frame}Q \approx \text{footprint}Q);\}}{\text{frame}Q \cap \{x\} = \emptyset} \rightarrow$

$Q_2$

$\{x \doteq 0, y \doteq 0\}$   
 $\{x \doteq 0, y \doteq 1\}$   
 $\{x \doteq 0, y \doteq 2\}$   
 $\{x \doteq 0, y \doteq 3\}$   
 $\vdots$

## Proving Universal Behavioral Properties

$x \doteq \text{old}(x)$  after executing  $Q(\text{frame}Q \approx \text{footprint}Q)$



# Modelling Original Calls as Abstract Statements



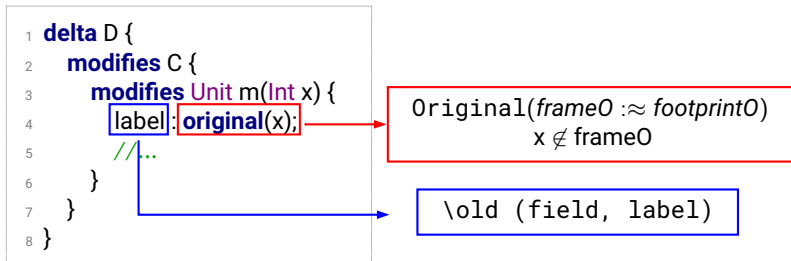
```
1 delta D {  
2   modifies C {  
3     modifies Unit m(Int x) {  
4       label : original(x);  
5       //...  
6     }  
7   }  
8 }
```

# Modelling Original Calls as Abstract Statements

```
1 delta D {  
2   modifies C {  
3     modifies Unit m(Int x) {  
4       label : original(x);  
5       //...  
6     }  
7   }  
8 }
```

Original( $frame0 \approx footprint0$ )  
 $x \notin frame0$

# Modelling Original Calls as Abstract Statements



# Verifying Deltas with Abstract Execution



```
1 class C {
2   Int f1;
3   Unit m(Int x){/*...*/}
4 }
5
6 delta D {
7   modifies C {
8     adds Int f2;
9     //@ensures f2 == \old(f2) + 1
10    //@    && f1 == \old(f1, l) + 1;
11    modifies Unit m(Int x) {
12      f2 = f2 + 1;
13      l : original(x);
14      f1 = f1 + 1;
15    }
16  }
17 }
```

# Verifying Deltas with Abstract Execution

```
1 class C {
2   Int f1;
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6 delta D {
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Original( $frameO \approx footprintO$ )  
 $x, f2 \notin frameO$   
 $f2 \notin footprintO$

# Verifying Deltas with Abstract Execution

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1 class C {
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Original( $frameO \approx footprintO$ )

$x, f2 \notin frameO$   
 $f2 \notin footprintO$

Beyond Liskov!

# Verifying Deltas with Abstract Execution

```
1 class C {
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6 delta D {
7   modifies C {
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10    //@    && f1 == \old(f1, l) + 1;
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15    }
16  }
17 }
```

Original( $frame0 \approx footprint0$ )  
 $x, f2 \notin frame0$   
 $f2 \notin footprint0$

This contract **cannot be invalidate** by delta applications



# Behavior Preserving Method



```
1 class C {
2   int f;
3
4   //@ensures this.f == \old(this.f) + 1;
5   Unit m1(){ f = f + 1;}
6
7   //@ensures this.f >= \old(this.f) + 3;
8   Unit m2() {
9     m1();
10    f = f + 2;
11  }
12
13  //@ensures this.f == \old(this.f) + 5;
14  Unit m3() {
15    m2();
16    f = f + 2;
17  }
18
19  //@ensures this.f == -1;
20  Unit m4(){ f = -1;}
21 }
```

```
1 delta D {
2   modifies C {
3
4     //@ensures this.f >= \old(this.f) + 7;
5     modifies Unit m2() {
6       original();
7       f = f + 4;
8     }
9
10    //@ensures this.f == 1;
11    modifies Unit m4() { f = 1;}
12
13  }
14 }
```

# Behavior Preserving Method



```
1 class C {
2   Int f;
3
4   //@ensures this.f == \old(this.f) + 1;
5   Unit m1(){ f = f + 1;} ← NOT MODIFIED
6
7   //@ensures this.f >= \old(this.f) + 3;
8   Unit m2() {
9     m1(); ← NOT MODIFIED
10    f = f + 2;
11  }
12
13  //@ensures this.f == \old(this.f) + 5;
14  Unit m3() {
15    m2();
16    f = f + 2;
17  }
18
19  //@ensures this.f == -1;
20  Unit m4(){ f = -1;}
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6
7   //@ensures this.f >= \old(this.f) + 3;
8   Unit m2() { ← LISKOV
9     m1(); ← NOT MODIFIED
10    f = f + 2;
11  }
12
13  //@ensures this.f == \old(this.f) + 5;
14  Unit m3() {
15    m2(); ← LISKOV
16    f = f + 2;
17  }
18
19  //@ensures this.f == -1;
20  Unit m4(){ f = -1; }
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```

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2   modifies C {
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4     //@ensures this.f >= \old(this.f) + 7;
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6       original(); ← LISKOV
7       f = f + 4;
8     }
9
10    //@ensures this.f == 1;
11    modifies Unit m4() { f = 1; }
12
13  }
14 }
```

```
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2   Int f;
3
4   //@ensures this.f == \old(this.f) + 1;
5   Unit m1(){ f = f + 1; } ← NOT MODIFIED
6
7   //@ensures this.f >= \old(this.f) + 3;
8   Unit m2() { ← LISKOV
9     m1(); ← NOT MODIFIED
10    f = f + 2;
11  }
12
13  //@ensures ??;
14  Unit m3() {
15    m4(); ← NON LISKOV
16    f = f + 2;
17  }
18
19  //@ensures this.f == -1;
20  Unit m4(){ f = -1; } ← NON LISKOV
21 }
```

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1 delta D {
2   modifies C {
3
4     //@ensures this.f >= \old(this.f) + 7;
5     modifies Unit m2() { ← LISKOV
6       original(); ← LISKOV
7       f = f + 4;
8     }
9
10    //@ensures this.f == 1;
11    modifies Unit m4() { f = 1; } ← NON LISKOV
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13  }
14 }
```

## Definition (Behavior Preserving Method)

A *Behavior Preserving Method*:

- does **not call non Behavior Preserving methods**
- is either
  - non **modified**
  - **modified** and **fulfilling Liskov**

## Definition (Behavior Preserving Sequence)

Sequence of statements **without calls to non Behavior Preserving methods**

## Definition (Behavior Preserving Method)

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- does **not call non Behavior Preserving methods**
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  - non **modified**
  - **modified** and **fulfilling Liskov**

## Definition (Behavior Preserving Sequence)

Sequence of statements **without calls to non Behavior Preserving methods**

⇒ Its behavior **cannot be modified** by delta application

# Behavior Preserving Method

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1 class C {
2   Int f;
3
4   //@ensures this.f == \old(this.f) + 1;
5   Unit m1(){ f = f + 1; } ← NOT MODIFIED
6
7   //@ensures this.f >= \old(this.f) + 3;
8   Unit m2() { ← LISKOV
9     m1(); ← NOT MODIFIED
10    f = f + 2;
11  }
12
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12
13  }
14 }
```



```
1 class C {
2   Int f;
3
4   //@ensures this.f == \old(this.f) + 1;
5   Unit m1(){ f = f + 1; } ← BP
6
7   //@ensures this.f >= \old(this.f) + 3;
8   Unit m2() { ← BP
9     m1(); ← BP
10    f = f + 2;
11  }
12
13  //@ensures ??;
14  Unit m3() { ← NON BP
15    m4(); ← NON BP
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```
1 class C {
2   Int f;
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5   Unit m1() { f = f + 1; } ← BP
6
7   // @ensures this.f >= \old(this.f) + 3;
8   Unit m2() { ← BP
9     m1(); ← BP ← BP-SEQUENCE
10    f = f + 2;
11  }
12
13  // @ensures ??;
14  Unit m3() { ← NON BP
15    m4(); ← NON BP
16    f = f + 2;
17  }
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5     modifies Unit m2() { ← BP
6       original(); ← BP
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9
10    // @ensures this.f == 1;
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12
13  }
14 }
```

# Shallow Single Original Normal Form

```
1 class C {
2   Int f1;
3   Unit m(Int x){/*...*/}
4 }
5
6 delta D {
7   modifies C {
8     adds Int f2;
9     //@ensures f2 == \old(f2) + 1
10    //@    && f1 == \old(f1, l) + 1;
11    modifies Unit m(Int x) {
12      f2 = f2 + 1;
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12      f2 = f2 + 1; ← BP-SEQUENCE
13      l : original(x);
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# Shallow Single Original Normal Form



```
1 class C {
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10    //@    && f1 == \old(f1, l) + 1;
11    modifies Unit m(Int x) {
12      f2 = f2 + 1; ← BP-SEQUENCE
13      l : original(x); ← original call
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# Shallow Single Original Normal Form



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12      f2 = f2 + 1;  $\Leftarrow$  BP-SEQUENCE
13      l : original(x);  $\Leftarrow$  original call
14      f1 = f1 + 1;  $\Leftarrow$  BP-SEQUENCE
15    }
16 }
17 }
```

## Definition

The body of  $\delta . m$  is  
 $\{S_1; \text{original}(\bar{p}); S_2;\}$

with  $S_1, S_2$  BP-sequences

## Refactorable

```
1 modifies Unit m(Int x) {  
2   if (x > 0) {  
3     field1 += 1;  
4     m1(); ← BP  
5     original(x);  
6   } else {  
7     original(x);  
8     field2 += 1;  
9   }  
10 }
```



## Refactorable

```
1 modifies Unit m(Int x) {  
2   if (x > 0) {  
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```

## Refactorable

```
1 modifies Unit m(Int x) {  
2   if (x > 0) {  
3     field1 += 1;  
4     m1();  $\leftarrow$  BP  
5     original(x)  
6   } else {  
7     original(x)  
8     field2 += 1;  
9   }  
10 }
```



## Refactored

```
1 modifies Unit m(Int x) {  
2   if (x > 0) {  
3     field1 += 1;  
4     m1();  $\leftarrow$  BP  
5   }  
6   original(x)  
7   if (x <= 0) {  
8     field2 += 1;  
9   }  
10 }
```



## Refactorable

```
1 modifies Unit m(Int x) {  
2   if (x > 0) {  
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## Refactored

```
1 modifies Unit m(Int x) {  
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3     field1 += 1;  $\Leftarrow$  BP-SEQUENCE S1  
4     m1();  $\Leftarrow$  BP  
5   }  
6   original(x);  $\Leftarrow$  original call  
7   if (x <= 0) {  
8     field2 += 1;  $\Leftarrow$  BP-SEQUENCE S2  
9   }  
10 }
```

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```
1 modifies Unit m(Int x) {
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5     original(x)
6   } else {
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7   if (x <= 0) {
8     field2 += 1;  $\Leftarrow$  BP-SEQUENCE S2
9   }
10 }
```

Using **Abstract Execution** to prove **correctness of refactoring**

# Change Old Behavior Principle

```
1 class C {
2   Int f1;
3   Unit m(Int x){/*...*/}
4 }
5
6 delta D {
7   modifies C {
8     adds Int f2;
9     //@ensures f2 == \old(f2) + 1
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12      f2 = f2 + 1;
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```

Original( $\text{frameO} \approx \text{footprintO}$ )

$x, f2 \notin \text{frameO}$   
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```

## Definition

The body of  $\delta . m$  is either:

- in normal form  
 $\{S_1; \mathbf{original}(\bar{p}); S_2;\}$   
where  $S_1$  assigns new fields, or
- a BP-sequence.

Original(frame0  $\approx$  footprint0)

$x, f2 \notin \text{frame0}$

$f2 \notin \text{footprint0}$

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1 class C {
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Original(frame0  $\approx$  footprint0)

$x, f2 \notin \text{frame0}$   
 $f2 \notin \text{footprint0}$

The behavior of  $\delta . m$  **cannot be affected** by delta application

# Theorem

The following theorem guarantees the correctness of this approach.

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*Assume the following:*

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- 1. Each method in the core is a BP-sequence, and verified.*

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- 2. For each  $\delta.m$  one of the following holds:*



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*Assume the following:*

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  - 2.1  $m$  is modified*

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## Theorem

*Assume the following:*

1. *Each method in the core is a BP-sequence, and verified.*
2. *For each  $\delta.m$  one of the following holds:*
  - 2.1  *$m$  is modified*
    - *fulfills Change Old Behavior Principle, and is verified, **or***

The following theorem guarantees the correctness of this approach.

## Theorem

*Assume the following:*

1. *Each method in the core is a BP-sequence, and verified.*
2. *For each  $\delta.m$  one of the following holds:*
  - 2.1 *m is modified*
    - *fulfills Change Old Behavior Principle, and is verified, or*
    - *is a BP-method, and is Liskov-verified*

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*Assume the following:*

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  - 2.1  *$m$  is modified*
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    - *is a BP-method, and is Liskov-verified*
  - 2.2  *$m$  is not modified (added), its body is a BP-sequence and is verified*

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*Assume the following:*

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  - 2.1  *$m$  is modified*
    - *fulfills Change Old Behavior Principle, and is verified, **or***
    - *is a BP-method, and is Liskov-verified*
  - 2.2  *$m$  is not modified (added), its body is a BP-sequence and is verified*
3. *No contract can call a non BP-method.*

The following theorem guarantees the correctness of this approach.

## Theorem

*Assume the following:*

1. *Each method in the core is a BP-sequence, and verified.*
2. *For each  $\delta.m$  one of the following holds:*
  - 2.1  *$m$  is modified*
    - *fulfills Change Old Behavior Principle, and is verified, or*
    - *is a BP-method, and is Liskov-verified*
  - 2.2  *$m$  is not modified (added), its body is a BP-sequence and is verified*
3. *No contract can call a non BP-method.*

*Then each variant is correct.*

---

# Delta-based Verification of Software Product Families

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**Normal Form** and **Abstract Execution** enable **delta-based verification**

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+

**original** is the only callable non BP-method



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↓

Deltas **cannot affect** the behavior of **non modified methods**

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**Normal Form** and **Abstract Execution** enable **delta-based verification**

+

**original** is the only callable non BP-method

⇓

Deltas **cannot affect** the behavior of **non modified methods**

⇓

Once a contract is proved it **cannot be invalidate** by delta applications

⇓

Each contract has to be **proven once**: **polynomial complexity  $O(N*M)$**   
( $N$ =nr. of deltas,  $M$ =nr. of methods)



## Expressiveness and Performance?



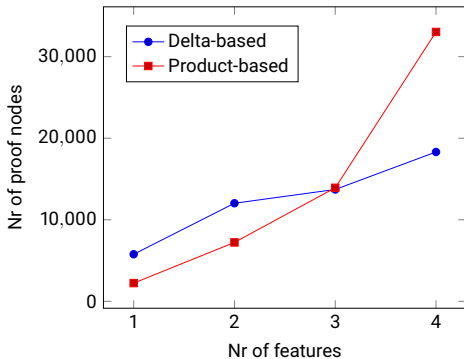
Specification and verification of

- **BankAccountSPL** Thüm et al. (2012)
- **MinePumpPL** Apel et al. (2013)

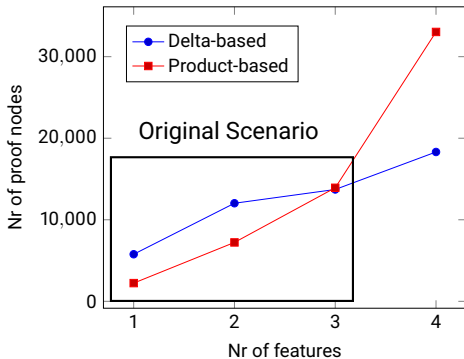


Specification and verification of

- **BankAccountSPL** Thüm et al. (2012)
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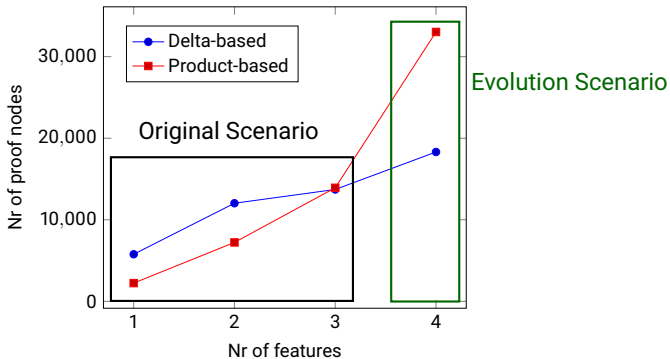


For a subset of **MinePumpPL**



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## Achievement

Overcome Liskov's restrictions *without increasing costs*

How?

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### How?

*Delta-based Verification:*  
**Normal Form**  $\Rightarrow$  **Abstract Execution**

**Expressiveness and Performance?**

## Achievement

Overcome Liskov's restrictions *without increasing costs*

## How?

*Delta-based Verification:*  
**Normal Form**  $\Rightarrow$  **Abstract Execution**

## Expressiveness and Performance?

- $\Rightarrow$  **Expressive** enough to specify existing product lines
- $\Rightarrow$  **Outperforms** product-based analysis, **suitable for evolving SPLs**

- 
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# Achieving Normal Form with AE



```
1 //before refactoring
2 modifies Unit m(p) {
3
4   if ( p1 == 0 ) {
5       field1 = 1;
6       m1(p1); //BP-method
7       original(p);
8       field2 +=1;
9   } else {
10      field1 = 2;
11      original(p);
12      m2(p2); //BP-method
13      field2 +=2;
14  }
15 }
```

↔

```
1 //after refactoring
2 modifies Unit m(p) {
3     Bool cond = p1 == 0;
4     if ( cond ) {
5         field1 = 1;
6         m1(p1); //BP-method
7     } else {
8         field1 = 2;
9     }
10    original(p);
11    if (cond) {
12        field2 +=1;
13    } else {
14        m2(p2); //BP-method
15        field2 +=2;
16    }
17 }
```

## Transformation rules:

1. Conditional Statement Condition Extraction (CSCE)
2. Unfold If Branch Prefix
3. Unfold Else Branch Prefix
4. Consolidate Duplicate Conditional Fragments (Extract Prefix)
5. Split Conditional Statement

```
if ( eboolean(frameE ≈ footprintE) ) {  
    Q1(frameQ1 ≈ footprintQ1);  
} else {  
    Q2(frameQ2 ≈ footprintQ2);  
}  
  
/*@ ae_constraint  
   @ x ∉ frameQ1 && x ∉ footprintQ1 &&  
   @ x ∉ frameQ2 && x ∉ footprintQ2;  
   @*/  
x = eboolean(frameE ≈ footprintE);  
if (x) {  
    Q1(frameQ1 ≈ footprintQ1);  
} else {  
    Q2(frameQ2 ≈ footprintQ2);  
}
```

CSCE